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A non-static elastic fluid distribution conformal to flat space–time

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Abstract. In this paper a non-static solution for an elastic fluid distribution has been obtained which is conformal to flat space–time.

1. Introduction

In our previous paper (Roy and Singh 1973), we gave a static solution for an elastic fluid distribution. The purpose of the present investigation is to obtain a non-static solution of an elastic fluid distribution which is conformal to flat space–time. A noteworthy characteristic of such solutions is that the conformal curvature tensor in this case vanishes.

We shall consider the conformally flat metric in the spherical-polar coordinate system, namely,

$$ds^2 = e^{\alpha(r,t)}(dr^2+r^2\,d\theta^2+r^2\sin^2\theta\,d\phi^2-dt^2). \tag{1}$$

The non-vanishing components of the energy–momentum tensor for (1) are given by

$$T_1^1 = -e^{-\alpha}\left(\frac{3}{4}\alpha'^2+2\frac{\alpha'}{r}\right)+e^{-\alpha}\left(\ddot{\alpha}+\frac{1}{4}\dot{\alpha}^2\right), \tag{2}$$

$$T_2^2 = T_3^3 = -e^{-\alpha}\left(\alpha''+\frac{\alpha'^2}{4}+\frac{\alpha'}{r}\right)+e^{-\alpha}(\ddot{\alpha}+\tfrac{1}{4}\dot{\alpha}^2), \tag{3}$$

$$T_4^4 = -e^{-\alpha}\left(\alpha''+\frac{\alpha'^2}{4}+2\frac{\alpha'}{r}\right)+e^{-\alpha}\frac{3}{4}\dot{\alpha}^2, \tag{4}$$

$$T_1^4 = -T_4^1 = e^{-\alpha}\left(\dot{\alpha}'-\frac{\dot{\alpha}\alpha'}{2}\right). \tag{5}$$

Einstein's interior field equations have been defined by Rayner (1963) for elastic bodies as follows:

$$R_{ij}-\tfrac{1}{2}Rg_{ij} = -\rho\lambda_i\lambda_j+\tfrac{1}{2}C_{ij}^{kl}(\mathfrak{g}_{kl}-\mathfrak{g}_{kl}^0) = -T_{ij}, \tag{6}$$

and the terms have their usual meaning as in our previous paper.

2. Solutions of the field equations

In a comoving coordinate system, we can satisfy the conditions of elastic body motion by taking $\mathfrak{g}_{ij}^0$ and C_{ij}^{kl} as functions of the space coordinates only. We regard them as known. v and μ are scalars and both are functions of r and t.

g^0_{ij} is the metric for an undeformed elastic body and hence we take it as flat, namely

$$g^0_{11} = 1, \qquad g^0_{22} = r^2, \qquad g^0_{33} = r^2 \sin^2\theta, \qquad g^0_{44} = -1. \tag{7}$$

The only nonzero surviving components of C_{ijkl} are C_{1111}, C_{1122}, C_{1133}, C_{1144}, C_{2222}, C_{2233}, C_{2244}, C_{3333}, C_{3344}, C_{4444}, and their values are

$$\begin{aligned}
C_{1111} &= \nu+2\mu,\\
C_{1122} &= \nu r^2,\\
C_{1133} &= \nu r^2 \sin^2\theta,\\
C_{1144} &= -\nu,\\
C_{2222} &= (\nu+2\mu)r^4\\
C_{2233} &= (\nu+2\mu)r^4 \sin^2\theta,\\
C_{2244} &= -\nu r^2,\\
C_{3333} &= (\nu+2\mu)r^4 \sin^4\theta,\\
C_{3344} &= -\nu r^2 \sin^2\theta,\\
C_{4444} &= \nu+2\mu.
\end{aligned} \tag{8}$$

Non-vanishing components of the energy–momentum tensor are given by

$$T^1_1 = \rho\lambda_1^2 e^{-\alpha} - \tfrac{1}{2}(\nu+2\mu)\lambda_1^2 e^{-\alpha} + \tfrac{1}{2}\nu\lambda_4^2 e^{-\alpha} + (2\nu+\mu)(e^{-\alpha}-1), \tag{9}$$

$$T^2_2 = T^3_3 = -\tfrac{1}{2}\nu\lambda_1^2 e^{-\alpha} + \tfrac{1}{2}\nu\lambda_4^2 e^{-\alpha} + (2\nu+\mu)(e^{-\alpha}-1), \tag{10}$$

$$T^4_4 = -\rho\lambda_4^2 e^{-\alpha} + (2\nu+\mu)(e^{-\alpha}-1) - \tfrac{1}{2}\nu\lambda_1^2 e^{-\alpha} + \tfrac{1}{2}(\nu+2\mu)\lambda_4^2 e^{-\alpha}, \tag{11}$$

$$T^4_1 = -T^1_4 = -\rho\lambda_1\lambda_4 e^{-\alpha}. \tag{12}$$

Hence, from equations (2)–(5) and (9)–(12), we have

$$-e^{-\alpha}\left(\tfrac{3}{4}\alpha'^2 + 2\frac{\alpha'}{r}\right) + e^{-\alpha}(\ddot{\alpha} + \tfrac{1}{4}\dot{\alpha}^2) = \rho\lambda_1^2 e^{-\alpha} - \tfrac{1}{2}(\nu+2\mu)\lambda_1^2 e^{-\alpha} + \tfrac{1}{2}\nu\lambda_4^2 e^{-\alpha} + (2\nu+\mu)(e^{-\alpha}-1), \tag{13}$$

$$-e^{-\alpha}\left(\alpha'' + \frac{\alpha'^2}{4} + \frac{\alpha'}{r}\right) + e^{-\alpha}(\ddot{\alpha} + \tfrac{1}{4}\dot{\alpha}^2) = -\tfrac{1}{2}\nu\lambda_1^2 e^{-\alpha} + \tfrac{1}{2}\nu\lambda_4^2 e^{-\alpha} + (2\nu+\mu)(e^{-\alpha}-1), \tag{14}$$

$$-e^{-\alpha}\left(\alpha'' + \frac{\alpha'^2}{4} + 2\frac{\alpha'}{r}\right) + \tfrac{3}{4}e^{-\alpha}\dot{\alpha}^2 = -\rho\lambda_4^2 e^{-\alpha} - \tfrac{1}{2}\nu\lambda_1^2 e^{-\alpha} + \tfrac{1}{2}(\nu+2\mu)\lambda_4^2 e^{-\alpha} + (2\nu+\mu)(e^{-\alpha}-1), \tag{15}$$

$$e^{-\alpha}\left(\dot{\alpha}' - \frac{\dot{\alpha}\alpha'}{2}\right) = -\rho\lambda_1\lambda_4 e^{-\alpha}. \tag{16}$$

The mathematical problem is to solve these four field equations containing five unknowns α, ρ, λ, ν and μ. But we expect that this problem could always be solved, just as in the modified theory of Einstein. Since there is one more unknown than equation, there will be one function which can be arbitrarily chosen. This function we choose as

$$\mu(r,t) = \rho(r,t) + \tfrac{1}{2}\left(r^2 - t^2 - 4\frac{t}{r}\right)e^{-rt}. \tag{17}$$

From (13) and (14), we get

$$e^{-\alpha}\left(\alpha''-\frac{\alpha'^2}{2}-\frac{\alpha'}{r}\right) = (\rho-\mu)\lambda_1^2 e^{-\alpha} \tag{18}$$

From (14) and (15), we get

$$e^{-\alpha}\left(\ddot{\alpha}-\frac{\dot{\alpha}^2}{2}+\frac{\alpha'}{r}\right) = (\rho-\mu)\lambda_4^2 e^{-\alpha} \tag{19}$$

From (18) and (19), we have

$$\alpha''-\frac{\alpha'^2}{2}-2\frac{\alpha'}{r} = \ddot{\alpha}-\frac{\dot{\alpha}^2}{2}+(\mu-\rho)e^{\alpha}. \tag{20}$$

It can be seen that

$$\alpha = rt \tag{21}$$

is a solution of (20).

Therefore, a non-static line element for an elastic fluid distribution is given by

$$ds^2 = e^{rt}(dr^2+r^2\,d\theta^2+r^2\sin^2\theta\,d\phi^2-dt^2), \tag{22}$$

the velocity vector λ_i of the distribution is given by

$$\lambda_i = \left\{\left[\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)e^{rt}\right]^{1/2}, 0, 0, \left[1+\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)e^{rt}\right]^{1/2}\right\}, \tag{23}$$

the density of distribution is given by

$$\rho = \tfrac{1}{2}(rt-2)\,e^{-rt}\left[\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)\right]^{-1/2}\left[1+\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)\right]^{-1/2} \tag{24}$$

and the scalars $v(r,t)$ and $\mu(r,t)$ are given by

$$v = \frac{\frac{1}{4}r^2-\frac{1}{4}t^2-t/r}{-\frac{3}{2}e^{rt}+2}-\frac{1-e^{rt}}{2-\frac{3}{2}e^{rt}}\left\{\frac{1}{2}(rt-2)e^{-rt}\left[1+\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)\right]^{-1/2}\right.$$
$$\left.\times\left[\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)\right]^{-1/2}\right\}+\tfrac{1}{2}(r^2-t^2-4t/r)e^{-rt}. \tag{25}$$

$$\mu = \tfrac{1}{2}(rt-2)e^{-rt}\left[1+\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)\right]^{-1/2}\left[\frac{2}{r^2-t^2-4t/r}\left(\frac{t^2}{2}+\frac{t}{r}\right)\right]^{-1/2}$$
$$+\tfrac{1}{2}(r^2-t^2-4t/r)\,e^{-rt}. \tag{26}$$

Thus a non-static generalization of our previous line element for an elastic fluid distribution has been obtained which is conformal to flat space–time.

References

Rayner C B 1963 *Proc. R. Soc.* A **272** 44–53
Roy S R and Singh P N 1973 *J. Phys.* A: *Math., Nucl. Gen.* **6** 1862–6